R. Society of Gottingen
from the Author

ON

THE APPLICATION OF MACHINERY

TO

THE COMPUTATION

OF

ASTRONOMICAL AND MATHEMATICAL TABLES.

By CHARLES BABBAGE, Esq. F.R.S.

From the Memoirs of the Astronomical Society of London.

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XXVI. *A Note respecting the Application of Machinery to the Calculation of Astronomical Tables.* By CHARLES BABBAGE, *Esq.* F.R.S. *Sec. Ast. Soc., &c. &c.*

Read 14th June, 1822.

IT is known to several of the members of this society that I have been engaged during the last few months in the contrivance of machinery, which by the application of a moving force may calculate any tables that may be required. I am now able to acquaint the society with the successful results at which I have arrived; and although it might at the first view appear a bold undertaking to attempt the construction of an engine which should execute operations so various as those which contribute to the formation of the numerous tables that are constantly required for astronomical purposes, yet to those who are acquainted with the method of differences the difficulty will be in a considerable degree removed.

I have taken the method of differences as the principle on which my machinery is founded; and in the engine which is just finished I have limited myself to two orders of differences. With this machine I have repeatedly constructed tables of square and triangular numbers, as well as a table from the singular formula x^2+x+41, which comprises amongst its terms so many prime numbers.

These, as well as any others which the engine is competent to form, are produced almost as rapidly as an assistant can write them down. The machinery by which these calculations are effected is extremely simple in its kind, consisting of a small number of different parts frequently repeated.

In the prosecution of this plan, I have contrived methods by which type shall be set up by the machine in the order determined by the calculation; and the arrangements are of such a nature that, if executed, there shall not exist the possibility of error in any printed copy of tables computed by this engine. Of several of these latter contrivances I have made models; and, from the experiments I have already made, I feel great confidence in the complete success of the plans I have proposed.

C. BABBAGE.

DEVONSHIRE-STREET, PORTLAND-PLACE,
June 2, 1822.

XXVII. *Observations on the Application of Machinery to the Computation of Mathematical Tables.* By CHARLES BABBAGE, Esq. F.R.S., &c. &c.

Read 13th December, 1822.

SINCE I had the honour of communicating to the Astronomical Society a short account of an arithmetical engine for the calculation of tables, which has been examined by several of the members of this society, I have not added much to the practical part of the subject. I have however paid some attention to the improvements of which the machinery is susceptible, and which will, if another engine is made, be greatly improved.

The theoretical inquiries to which it has conducted me are however of a singular nature; and I shall take this opportunity of briefly explaining to the society some of the principles on which they depend, as far as the nature of the subject will permit me to do this without the introduction of too many algebraic operations, which are rarely intelligible when read to a large assembly.

Of the variety of tables which are required in the present state of science, by far the larger portion are intimately connected with that department of it which it is the peculiar object of this society to promote.

The importance of astronomical science, whether viewed as the proudest triumph of intellectual power, or considered as the most valuable present of abstract science to the comfort and happiness of mankind, equally claims for it the first assistance from any new method for condensing the processes of reasoning or abridging the labour of calculation. Astronomical tables were therefore the first objects on which I turned my attention, when attempting to improve the power of the engine, as they had formed the first motive for constructing it.

I have already stated to the society, in my former communication, that the first engine I had constructed was solely destined to compute tables having constant differences. From this circumstance it will be apparent that after a certain number of terms of a table are computed, unless, as rarely happens, it has a constant order of differences, we must stop the engine and place in it other numbers, in order to produce the next portion of the table. This operation must be repeated more or less frequently according to the nature of the table. The more numerous the order of differences, the less frequent will this operation become requisite. The chance of error in such computations arises from incorrect numbers being placed in the engine: it therefore becomes desirable to limit this chance as much as possible. In examining the analytical theory of the various differences of the sine of an arc, I noticed the property which it possesses of having any of its even orders of differences equal to the sine of the same arc increased by some multiple of its increment multiplied by a constant quantity. With the aid of this principle an engine might be formed which would require but little attendance, and I believe that it might in some cases compute a table of the form A sin θ from the 1st value of $\theta=0$ up to $\theta=90°$ with only one set of figures being placed in it.

It is scarcely necessary to observe what an immense number of astronomical tables are comprised under this form, nor the great accuracy which must result from having reduced to so few a number the preliminary computations which are requisite.

In pursuing into its detail the principle to which I have alluded, which lends itself so happily to numerical application, I have traced its application to other species of tables, and am enabled to point out a course of analytical investigation which will in all probability afford ready methods for constructing tables, even of the most complicated transcendent, in a manner equally easy.

I will now advert to another circumstance, which, although not immediately connected with astronomical tables, resulted from an examination of the engine by which they can be formed.

On considering the arrangement of its parts, I observed that a different mode of connecting them would produce tables of a new species altogether different from any with which I was acquainted. I therefore computed with my pen a small table such as would have been formed by the engine had it existed in this new shape, and I was much surprised at discovering that no analytical method was yet known for determining its n^{th} term. The following is the first series I wrote down:

	Series.	Diff.		Series.	Diff.		Series.	Diff.
0 . .	2	0	10 . . .	222	42	20 . . .	924	86
1 . .	2	2		264	46		1010	86
	4	6		310	46		1096	92
	10	6		356	52	25 . . .	1188	100
	16	12	15 . . .	408	60		1288	108
5 . . .	28	20		468	68		1396	114
	48	28		536	74		1510	114
	76	34		610	74		1624	118
	110	34		684	78	30 . . .	1742	120
	144	38	20 . . .	762	80		1862	122
10 . .	182	40		842	82		1984	

The equation of finite differences from which it is produced is

$$\Delta^2 u_z = \text{units fig. of } u_{z+1}$$

which is one of a class of equations never hitherto integrated. I succeeded in transforming this equation into a more analytical form: but still it presented great difficulties; I therefore undertook the investigation in a different manner, and succeeded in discovering a formula which represented its *n*th term. It is the following:

$$u_z = (\bar{a}) + 206\,(106 + 2a - 1)$$

TABLE

0	2
1	2
2	4
3	10
4	16
5	28
6	48
7	76
8	110
9	144

where $(\bar{a})$ represents the number opposite a in the annexed subsidiary table, and a is the figure in the unit's place of z, and b is that number which arises from cutting off the last figure from z. Example: let the 17th term be required, then $z=17$, and $a=7$, $b=1$; the number opposite 7 in the table is

$$(\bar{7}) = \quad . \quad . \quad . \quad 76$$

$$106 + 2a - 1 = 10 + 14 - 1 = 23$$

$$206 = 20 \qquad 206\,(106 + 2a - 1) = \underline{460}$$

$$536 = u_{17}$$

I have formed other series of the same class, and have succeeded in expressing any term independent of all the rest by two distinct processes. Thus I have incidentally been able to integrate the equations I have mentioned: I will just state one other of a simple form; it is the equation

$$\Delta u_z \text{ units fig. } u_z$$

whose integral is

$$u_z = 20b + 2^a$$

where a is that one of the numbers 1, 2, 3, 4, which taken from z leaves the remainder divisible by 4, and b is the quotient of that division.

The table is as follows:

1	–	2
		4
		8
4	–	16
		22
		24
		28
8	–	36
		42

One of the general questions to which these researches give rise is, supposing the law of any series to be known, to find what figure will occur in the kth place of the nth term. That the mere consideration of a mechanical engine should have suggested these inquiries, is of itself sufficiently remarkable; but it is still more singular, that amongst researches of so very abstract a nature I should have met with and overcome a difficulty which had presented itself in the form of an equation of differences, and which had impeded my progress several years since, in attempting the solution of a problem connected with the game of chess.

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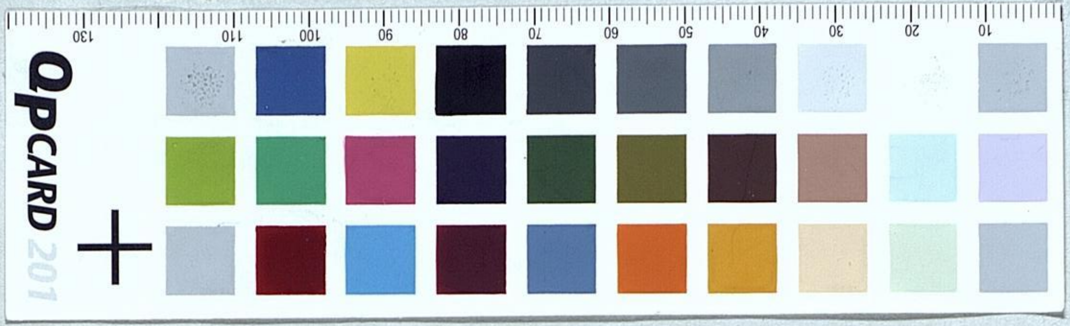
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